

R18

Code No: 154AQ

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year II Semester Examinations, December – 2024/January - 2025

DISCRETE MATHEMATICS

(Common to CSE, IT, ITE, CE(SE), CSE(CS), CSE(N))

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

- 1.a) Prove by induction that the sum of the first n odd numbers is n^2 . [2]
- b) Construct the truth table for $p \rightarrow (\neg q \vee r)$. [3]
- c) Represent the relation $R = \{(1,2), (2,3), (3,1)\}$ on the set $\{1,2,3\}$ using a matrix. [2]
- d) Find the sum of the first 8 terms of the geometric sequence where the first term is 1 and the common ratio is 2. [3]
- e) Determine the Big-O notation for the following function: $f(n) = 3n^2 + 2n + 1$. [2]
- f) Prove that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ using mathematical induction. [3]
- g) If two fair dice are rolled, what is the probability of getting a sum of 7? [2]
- h) In a group of 40 students, 25 like chocolate, 20 like vanilla, 15 like strawberry, 10 like both chocolate and vanilla, 5 like both vanilla and strawberry, 8 like both chocolate and strawberry, and 3 like all three. How many students like at least one of the three flavors? [3]
- i) Represent the graph $G = (V, E)$ where $V = \{1, 2, 3\}$ and $E = \{(1, 2), (2, 3)\}$ using an adjacency matrix. [2]
- j) Determine if the graph $G = (V, E)$ where $V = \{A, B, C, D\}$ and $E = \{(A, B), (B, C), (C, D), (D, A), (A, C)\}$ is planar or not. Justify your answer. [3]

PART – B

(50 Marks)

- 2.a) Draw the gating network of the formula $(p \downarrow s) \wedge (\neg p \vee (q \rightarrow r)) \vee (p \vee q \vee s)$.
- b) Consider the following statements:

“If you send me the text message, then I will finish the assignment,”

“If you do not send me the text message, then I will go to sleep early,” and

“If I go to sleep early, then I will wake up feeling guilty”.

Using the rules of inferences show that above premises lead to the conclusion “If I do not finish the assignment, then I wake up feeling guilty”.

[3+7]

OR

3. Use rules of inference to show that if $(\forall x)(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $(\forall x)(P(x) \wedge R(x))$ are true then $(\forall x)(R(x) \wedge S(x))$ is true, where the domains of all quantifiers are the same.

[10]

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4. Let D_n , be the set of all divisors of n . Draw the Hasse diagram of the Lattice D_n under the relation divisibility for $n = 30, 72$.

- a) Which of these lattices are complemented? Justify your answer.
 b) Which of these lattices are distributive? Justify your answer.
 c) List any two sublattices of length 4 from each of these two lattices.
 d) List any one subset which is not a sublattice from each of these two lattices.

[2+3+3+2]

OR

5. Let f and g be functions from $\{1,2,3,4\}$ to $\{a,b,c,d\}$ and from $\{a,b,c,d\}$ to $\{1,2,3,4\}$, respectively, with $f(1) = d, f(2) = c, f(3) = a$, and $f(4) = b$, and $g(a) = 2, g(b) = 1, g(c) = 3$, and $g(d) = 2$.

- a) Is f one-to-one? Is g one-to-one?
 b) Is f onto? Is g onto?
 c) Does either f or g have an inverse? If so, find this inverse.

[3+3+4]

6. Give a big- O estimate for $f(x) = (x + 1)\log(x^2 + 1) + 3x^2$.

[10]

OR

7. Use mathematical induction to show that $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$ for all non negative integers n .

[10]

8. Suppose that one person in 100,000 has a particular rare disease for which there is a fairly accurate diagnostic test. This test is correct 99.0% of the time when given to a person selected at random who has the disease; it is correct 99.5% of the time when given to a person selected at random who does not have the disease. Given this information can we find

- (a) the probability that a person who tests positive for the disease has the disease?
 (b) the probability that a person who tests negative for the disease does not have the disease?

- (c) Should a person who tests positive be very concerned that he or she has the disease?

[3+3+4]

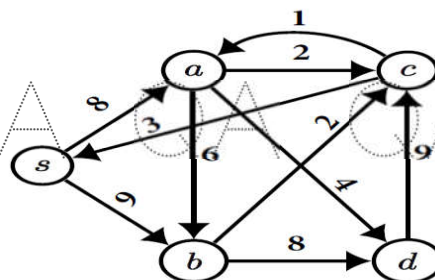
OR

9. How many solutions does $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 , and x_3 are nonnegative integers with $x_1 \leq 3, x_2 \leq 4$, and $x_3 \leq 6$?

[10]

10. Find the all-pair shortest path rooted at the vertex s of the directed graph 1 shown below:

[10]



Graph 1

OR

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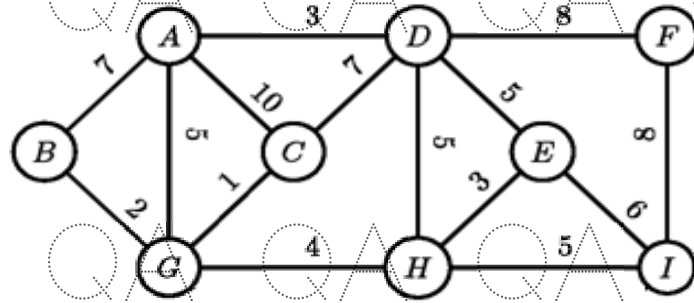
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11. Find the minimum spanning tree for the weighted graph 2 given below using Prim's algorithm. Provide the step-by-step workout. [10]

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Graph 2

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